Photon Trajectory Attributes of an Expanding Hypersphere

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Abstract
Analytical expressions are derived to characterize the photon trajectory attributes of an expanding hyperspherical Friedmann–Lemaître framework. The results imply the existence of distortion effects that can significantly impact a local observer's view of distant objects in terms of the object's observed location, distance, size, and time flow in such a framework. The effects of such distortion on cosmological observability, cosmic background radiation, and quasar observations are reviewed. It is demonstrated that the nature of the analytically predicted observed field of view is one that not only satisfies the requirements of the well-known Hubble rule for distance and recession velocity, but also predicts observed redshift values in excess of $z = 1$. *The impact of photon trajectory on observed cosmic background radiation temperature, on the other hand, suggests the need for a* $z \geq 11$ *redshift correction. The derived solution for distortion can also be viewed as logically equivalent to one for the general relativistic local framework space-time curvature caused by the expansion of the hypersphere. The results of this solution are compared with those of a conventional special relativity-based model commonly employed to estimate quasar distance and age. Distance estimates from both solutions are shown to be remarkably close* ($0 \leq$ *error* $< 8\%$). *Age estimates, however, although close in value for observed redshift below* $z = \pi/2$, *tend to diverge strongly for* $z > \pi/2$.

Key words: photon trajectory, cosmological distortion, lens effect, expanding hypersphere, Friedmann–Lemaître framework, global model, light cone, redshift, Hubble rule, recession velocity

1. INTRODUCTION

The contents of this paper focus on the subject of how a dynamically expanding hyperspherical space-time framework can cause a photon to follow a curved trajectory rather than the straight-line path obtained from flat space-time representations of light propagation. Intuitively, such a dynamic model can be viewed as characterizing the big bang event globally in terms of an "expansion of space" process. The presentation extends initial results on the same subject given in an earlier paper by the authors at the GR13 conference.[1]

As might be expected, the hyperspherical framework described here bears a close relationship to the global modeling work of Penrose and Hawking[2-7] for light cones in both black hole and Friedmann–Lemaître expansion system environments. However, whereas a significant part of that work emphasizes the treatment of singularities, the expanding hypersphere employed here instead focuses on representing such a model dynamically and geometrically in terms of a derived set of exterior–perspective space-time properties. Given such a definition of space-time, some obvious questions to be considered are the following:

(1) What are the fundamental properties of this space-time?
(2) What does it imply about photon trajectory?
(3) To what extent do cosmological observations support the notion of such a definition of space-time?
(4) What is its relationship to "standard model" cosmology?

The presentation given in the following will attempt to answer these questions.

The contents of this paper will also call to mind some black hole photon trajectory computer simulation traces attributed to Dicke.[8] These simulations illustrate, for example, how a photon trajectory, when defined inside the Schwarzschild radius of a black hole, will follow an inward spiraling curved path toward the origin of the black hole. Interpreted within the context of the Hawking doctoral thesis work, this would suggest an outward spiraling photon trajectory for an expanding hyperspherical framework. Such a viewpoint anticipates one of the principal analytical results presented in this paper.

If one is willing to tentatively accept the existence of a spiraling trajectory for photons in an expanding hypersphere, this then implies the existence of a definable "lens effect" in cosmological observations, especially for very distant objects. Such an effect, following Einstein, was predicted by Klauder *et al.*[9] Their work indicates that

> curved space should act as a lens of great focal length. Distant galaxies — from a quarter to halfway around the

universe — are expected to have greatly magnified angular diameters, especially at redshift factor $z = 2$ or more.[9]

New results given in this paper will dwell at length upon the details of such a "lens effect" in an expanding spherically connected framework. It should also be noted that much of the reference material cited thus far is summarized in the well-known book on gravitation by Misner, Thorne, and Wheeler.[10]

The presentation given here is organized as follows: First, using fundamental properties of light cones found in an expanding 4-space hyperspherical space-time system, a simplified 2-space model (based on well-known great circle geodesic path concepts) is defined as a vehicle for analyzing photon trajectory. Next, an analytical result for photon trajectory locus is developed and analyzed to determine local observational impacts on distance, size, time flow, and redshift of an observed distant object. Discussion is then pursued regarding the impact of photon path curvature in an expanding hypersphere on cosmological observability, big bang background radiation observations, and quasar observations.

Use of a Penrose–Hawking global model baseline allows one to associate the results given here with the local general relativity solutions for Friedmann–Lemaître expansion systems underlying such global models.. The derived results for "lens effect" distortion can therefore be viewed as logically equivalent to relativistic solutions regarding locally observed space-time transformations in an expanding hyperspherical framework. This implies that complex local analysis methods may be exchanged for a simplified solution approach which exploits the topological properties of an implied exterior perspective space-time structure.

2. MODEL DEFINITION

The global model employed here is one that makes use of time-oriented causally connected event horizons in an expanding hyperspherical space-time framework. The model may be viewed as an implied topological derivative of the global models developed by Penrose and Hawking.[2-7] One particular feature of the model concerns the use of an exterior perspective space-time manifold which is explicitly spherical. The basis for this choice stems from

(1) global black hole analysis results,[11,12] which indicate that topologies other than spherical cannot occur;
(2) global analysis,[7] which establishes the general applicability of black hole modeling results to an oppositely expanding Friedmann–Lemaître framework.

Thus space-time has been altered from its usual local framework flat space-time definition to be defined instead in terms of an expanding 4-space hyperspherical structure. For the sake of simplification, it is also assumed that the hyperspherical structure is expanding radially along its temporal (imaginary) axis relative to an initial state at a linear rate c, the velocity of light in free space.

Because of the time-oriented nature of the model, each event in space-time is uniquely associated with a future and past light cone corresponding to a path locus defined by a set of radial null geodesics in expanding hyperspherical space-time. The locus of this path must not only continuously span a set of causally connected events in space-time, but must also conform to the specific boundary conditions dictated by the topology of its environment, both globally and locally. Thus all light cones must locally conform to alignment constraints dictated by general relativity for locally perceived Minkowski flat space-time. This condition will be invoked in the next section to develop an analytical expression for photon path locus.

A two-dimensional diagram for the space-time model used here is shown in Fig. 1. The circles of the diagram correspond to hyperspheres of relative simultaneity for events in an expanding set of horizons. Thus time is explicitly represented in the model as a set of (imaginary axis) linear flow radials (aligned locally with the Minkowski flat space-time imaginary temporal axis) that causally connect the events of the horizon space. Since the linear flow of time and the expansion rate of the event horizon hypersphere are specifically associated with the constant c, a simple metric, ct, can be invoked to define the locus of the now-hypersphere as a function of time. The spherical nature of the space-time model shown in Fig. 1 also conveniently allows one to define spatial locus as a function of angular measure. These variables (in time and angular location) can then be used to support development of an analytical expression for photon path locus in an expanding hypersphere. As indicated in Fig. 1, the particular photon path locus of interest in this paper is that which lies between an observer's now-hypersphere location at angle θ_2 and that of a distant object on a past hypersphere at θ_1. These angles are defined relative to a selected common origin reference. The angle $\phi = \theta_2 - \theta_1$ then is used to define the angular locus of the observed object relative to the observer's point of reference location on the now-hypersphere.

It will be seen that the metric associated with distance between the observer and the observed object can be defined in 2-space as

$$ds^2 = c^2[t^2 + (\exp \phi)^2]/2 \, d\phi^2, \tag{1}$$

because the shortest photon path between observer and object must lie uniquely on the expanding "great circle" set of arcs defined by the angle ϕ. Thus photon trajectory solutions that correspond to local general relativity space-time transformations can be developed with significantly less analytical effort than that required by direct use of local 4-space general relativity analysis methods.

3. GLOBAL ANALYSIS

An exact solution for photon trajectory in the expanding spherically connected framework can be derived by evaluating the locus of the light cone perimeter. By virtue of the geodesic properties found in a hypersphere, such an evaluation can be made using the two-dimensional space-time model shown in Fig. 1. As a necessary condition, each edge of the light cone perimeter must define a continuous well-behaved path that inter-

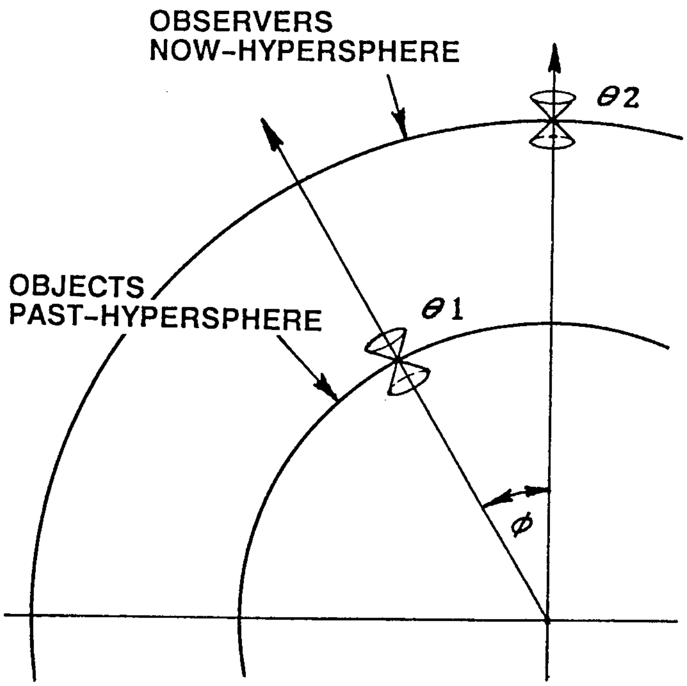

Figure 1. Two-dimensional view of expanding hyperspherical framework.

sects the expanding event horizon circle locally at an angle of 45°. This follows from a constraint that requires locally perceived flat space-time. One might also consider defining a constraint regarding preservation of trajectory path length, but, as will be demonstrated, this is unnecessary.

Mathematically, the constraint condition for a 45° intersection can be represented as shown in Fig. 2. The element dr shown in Fig. 2 represents trajectory path length radially, and dq represents an element of length in a direction orthogonal to that of the radial component. In equation form the constraint requires that $dq = dr$, $rd\theta = cdt$. This in turn yields photon path locus expressions

$$\theta = \int \frac{c}{r} dt = \int \frac{1}{t} dt = \ln \frac{r}{c} + k = \ln t + k, \tag{2}$$

where k is a constant of integration which defines the origin of θ as a function of the system of units used in metrics for time t and distance r. The equation forms in (2) may then be rearranged to obtain

$$r = c \exp(\theta - k), \quad t = \exp(\theta - k) \tag{3}$$

as an alternate form which also defines the locus of the light cone perimeter. Several convenient values of k correspond to

(1) sidereal time in seconds, which forces $\theta = 0$ at one second after the initiation of the hyperspherical expansion process, and
(2) Planck-based units of time T^* or distance L^*, which force $\theta = 0$ at a reference point such that physical processes involving photon trajectories are not in conflict with currently understood quantum mechanical constraints.

For the most part, however, the analysis presented in this paper will focus on $\phi = \theta_2 - \theta_1$, and therefore the need for explicit definition of an origin reference for θ will be avoided.

As illustrated in Fig. 3, the expression in (3) defines a photon trajectory locus which is an outward logarithmic spiral. Such an outward spiral can be viewed as a time-reversed analog of the photon trajectory traces that have been developed for the region inside a black hole that follow an inward spiraling trajectory.[8] A more complete representation of the light cone perimeter topology is shown in Fig. 4. It can be seen from this diagram that the expanding hypersphere causes the past light cone to converge exponentially as one looks in the distance and back in time toward the origin of the hypersphere. The future light cone, on the other hand, diverges exponentially as expansion continues beyond the locus of the now-hypersphere. Because all trajectories spiral outward, one sees only the past. The curvature of the past light cone locus indicates that there will be significant "lens effect" distortion[9] affecting the space-time appearances of distant objects. This point will now be examined in detail.

It was mentioned that no constraint on the preservation of photon trajectory path length was needed. The question to be answered is, What is the length of the curved path s shown in Fig. 3 which joins the observed object on a past-hypersphere at locus $\theta 1$ to that of an observer on the now-hypersphere at locus θ_2? Moreover, how does the length of s compare with s', the perimeter of the light cone defined in flat space-time? Using the metric in Eq. (1), the length of s is

$$s = \int_{\theta_1}^{\theta_2} ds = \int_{\theta_1}^{\theta_2} c \left[x^2 + \left(\frac{dx}{d\theta} \right)^2 \right]^{1/2} d\theta , \tag{4}$$

where $x = r/c = t$ and $dx/d\theta = \exp \theta$. Evaluating further, one obtains

$$s = 2^{1/2} c \int_{\theta_1}^{\theta_2} \exp \theta d\theta = 2^{1/2} c(\exp \theta_2 - \exp \theta_1) = s' , \tag{5}$$

which demonstrates that the length of the spiraling path is the same as that which would have occurred in conventional flat space-time. It can be seen from Eq. (3), assuming $k \to 0$, that

$$\exp \theta_2 = T, \tag{6}$$

where T is the age of the now-hypersphere, and

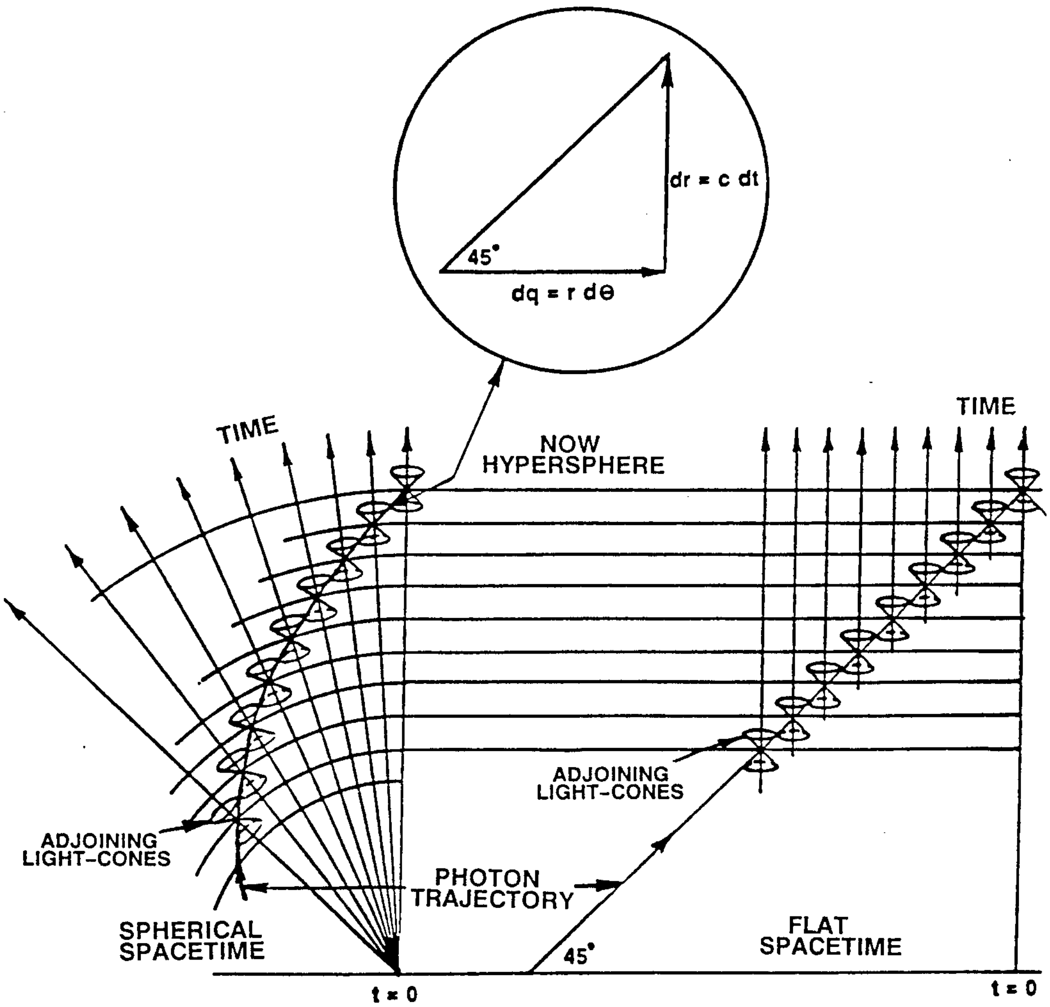

Figure 2. Time cone locus constraint condition.

$$\exp\theta_1 = T\exp(-\phi). \tag{7}$$

Therefore, one obtains

$$s = 2^{1/2} cT[1 - \exp(-\phi)] \tag{8}$$

as a compact alternative representation of trajectory path length.

The ratio of trajectory path length to that of the actual distance between the observer and the object at relative angle ϕ is simply $2^{1/2}$. Thus, as implied by the trajectory path length in Eq. (8), the actual distance is

$$d = cT[1 - \exp(-\phi)]. \tag{9}$$

The size and time flow attributes of the observed object at relative angle ϕ are also distorted as a function of ϕ. This may be seen by analyzing the derivatives of length and time with respect to relative angle ϕ:

$$dr/d\phi = c\exp(\theta_2)\exp(-\phi) = c\exp(\theta_1)\exp(\phi), \tag{10}$$

$$dt/d\phi = \exp(\theta_2)\exp(-\phi) = \exp(\theta_1)\exp(\phi). \tag{11}$$

The form of these expressions demonstrates that the measures of observed length and time flow for the object at ϕ are enlarged by a factor $\exp\phi$. This enlargement is illustrated in Fig. 5, where the local span between the two photon trajectories leading to the now-hypersphere diminishes exponentially as the relative angular separation between observer and object increases. Object length enlargement also causes the observed distance to the object to be enlarged as indicated by

$$D = c\int_{\theta_1}^{\theta_2} \exp(\theta_1)\exp(\phi)\,d\theta = c\exp\theta_2(\theta)\Big|_{\theta_1}^{\theta_2} = c\phi T, \tag{12}$$

which can be derived from Eqs. (6) and (10) after noting $d\theta = d\phi$. This expression indicates that the observed object appears to be projected onto the vicinity of the now-hypersphere at a relative angle of ϕ. The observed object therefore appears to be moving from the observer with an apparent recession velocity $v = c\phi = cz$, which implies that the observed redshift factor due to expansion of the hypersphere is

$$z = \phi \text{ rad.} \tag{13}$$

A quantitative representation of the distortion impacts caused by photon trajectory curvature is shown in Table I. The table indicates that simple linear rules of measure hold only at distances that are relatively close to the observer. Beyond one's immediate galaxy, significant adjustments to the size, distance, and time flow features of an observed object are required to obtain correct information about the local features of an observed object. This point is further explored in the next section, where some specific examples involving cosmological observation on distant objects are considered.

Table I: Distortion Impact on Observations

Observed Redshift ϕ *rad*	*Length/Time Enlargement* exp (ϕ)	*Actual Distance* $cT[1 - 1/\exp(\phi)]$	*Observed Distance* $cT\phi$
0	0.0	0	0
1	2.7	$0.63cT$	cT
$\pi/2$	4.8	$0.79cT$	$\pi cT/2$
π	23	$0.96cT$	πcT
2π	535	$0.998cT$	$2\pi cT$
3π	12391	$0.9999cT$	$3\pi cT$
4π	286751	$0.999996cT$	$4\pi cT$
$\sim 13\pi$	$t = 1$ s		
$\sim 44\pi$	$t = T^*$ (Planck time)		
$\to\infty$	$t \to 0$		

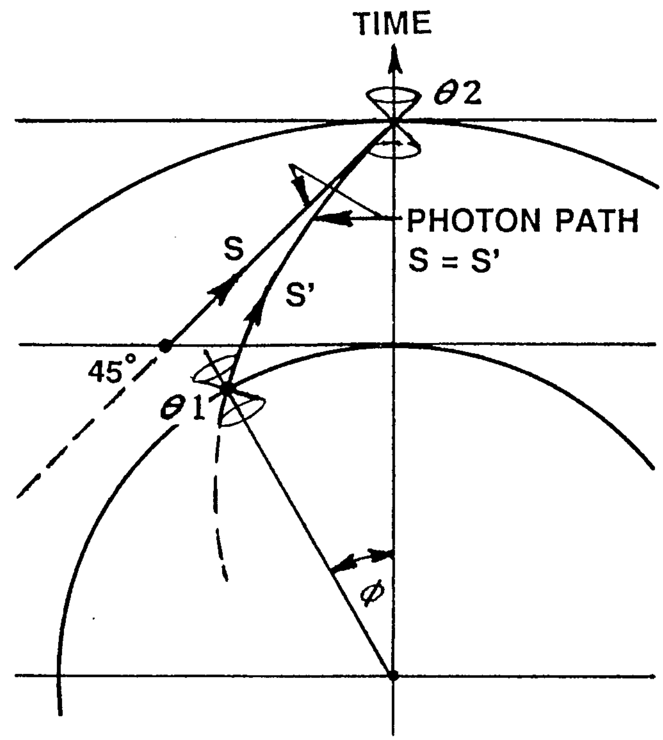

Figure 3. Curved photon trajectory defined by light cone perimeter.

4. DISCUSSION

4.1 Cosmological Observability

From a cosmological observability viewpoint, the fundamental nature of the distortion caused by photon trajectory curvature might loosely be described as Mercator-projection-like. However, Mercator projection simply regards distortions that arise from mapping a two-dimensional spherical surface of finite radius to a cylindrical surface. The distortion encountered by a cosmological observer, on the other hand, involves mapping the observer's outward diverging spherical field of view to an inward converging spherical objective field. This point is illustrated in Fig. 6. As shown, all lines of sight that can be observed must map into a collapsing hyperspherical system and tend to terminate at its origin as one looks far enough in the distance (back in time). Such a mapping requirement provides a geometric/physical basis for the "lens effect" distortions described by the analytical expressions given in Eqs. (9) to (13). In the limit, the singularity at the origin of the expanding system must map to the outer limit of the observer's field of view. Thus, except for quantum mechanical barriers at $t = T^*$ (Planck time) or lengths and distances equal to L^* (Planck length), the observer's line of sight would appear to be unbounded.

The spiraling photon path illustrated in Figs. 3 and 4, however, also gives rise to a "many to one" mapping form of cosmological observability. That is to say, along each line of sight, the same distinct regions of the hypersphere that are observed as ϕ traverses the region $0 \leq \phi \leq 2\pi$ are observed again cyclically at an earlier stage of development as ϕ continues to cyclically traverse each succeeding range of 2π rad. Each "epoch" corresponding to a range of 2π rad in ϕ appears as a segment on the line of sight with an observed length equal to $2\pi cT$. This is shown graphically in Fig. 7. In theory, an observer could see a view of his own region on a past-hypersphere in the distance at $\phi = 2\pi$ if it were somehow luminous and not blocked from sight by luminal and/or gravitational interference from the nearby components of his own immediate galaxy. As suggested by Eqs. (12) and (13), an object at that location would appear to be projected on the vicinity of the now-hypersphere with redshift $z = 2\pi$.

It should also be noted that the Hubble rule regarding recession velocity and distance is in exact agreement with the observed distance expression in Eq. (12) as well as the interpre-

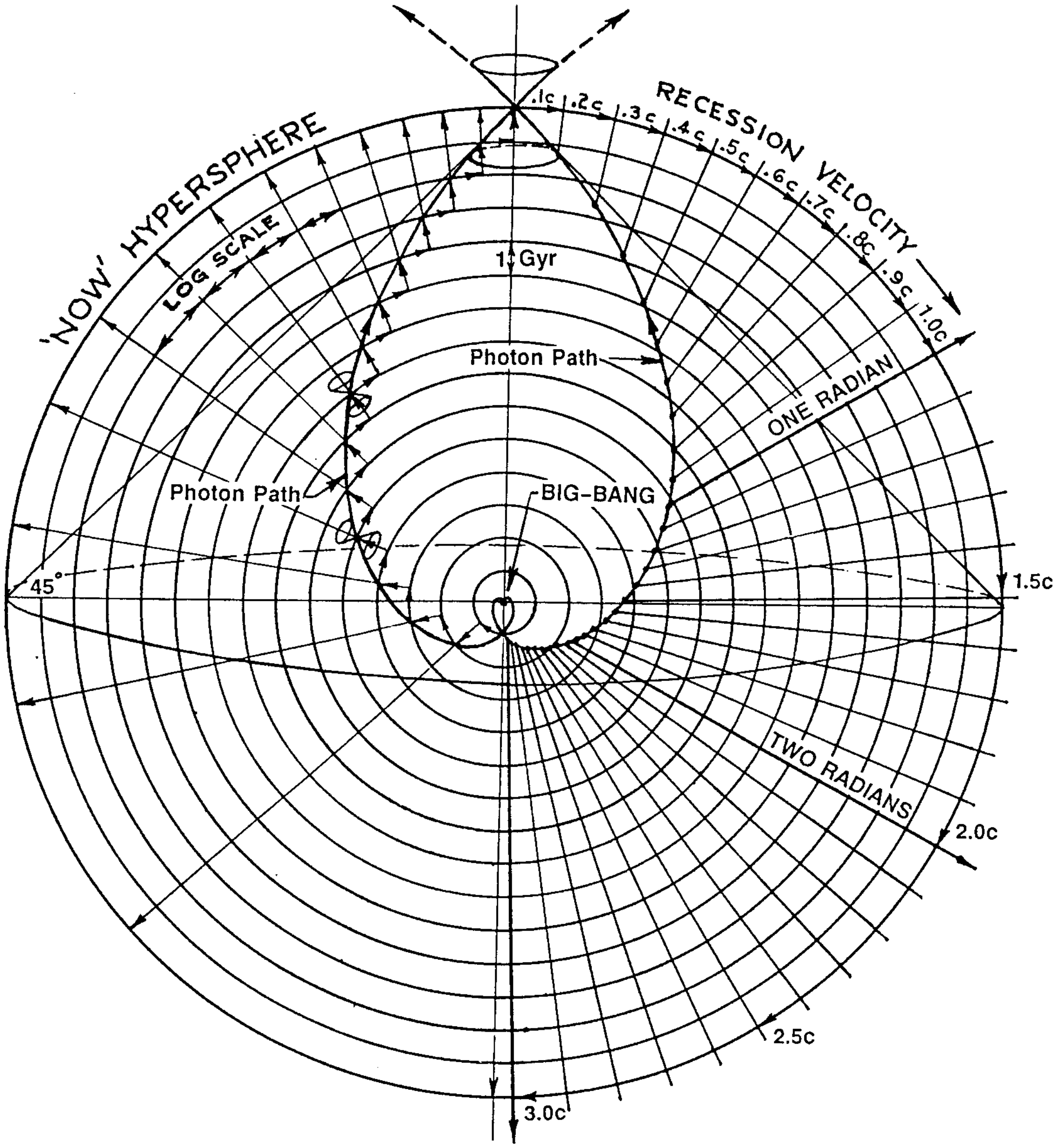

Figure 4. Global light cone defined as a logarithmic spiral.

tation given in connection with it in the prior paragraph. According to Eqs. (12) and (13), an observer's apparent view of distant objects (in the past) places them in the vicinity of the now-hypersphere. As the hypersphere continues to expand, observed objects appear to recede at a velocity that is exactly proportional to their observed distance. Thus the derived results not only satisfy the Hubble rule, but also, as shown in Eq. (13), provide a basis for observed redshift values above $z = 1$. Conventional flat space-time models with straight-line photon trajectories, on the other hand, inherently lack a mechanism that allows observed redshift to exceed $z = 1$, without implying an observed relative velocity above the speed of light c. Thus the vast amount of data regarding high redshift quasars with $z > 1$ suggests the need for a space curvature mechanism such as that provided by the expanding hyperspherical space-time model described in this paper.

Regarding observability, one is also strongly tempted to use the word "inflationary" to describe the nature of the distortions caused by photon trajectory curvature. We speak here, in particular, of the exponential "lens effect" distortion implied by Eqs. (10) and (11). An observer who is unaware of such distortion might be persuaded to define a theoretical basis for

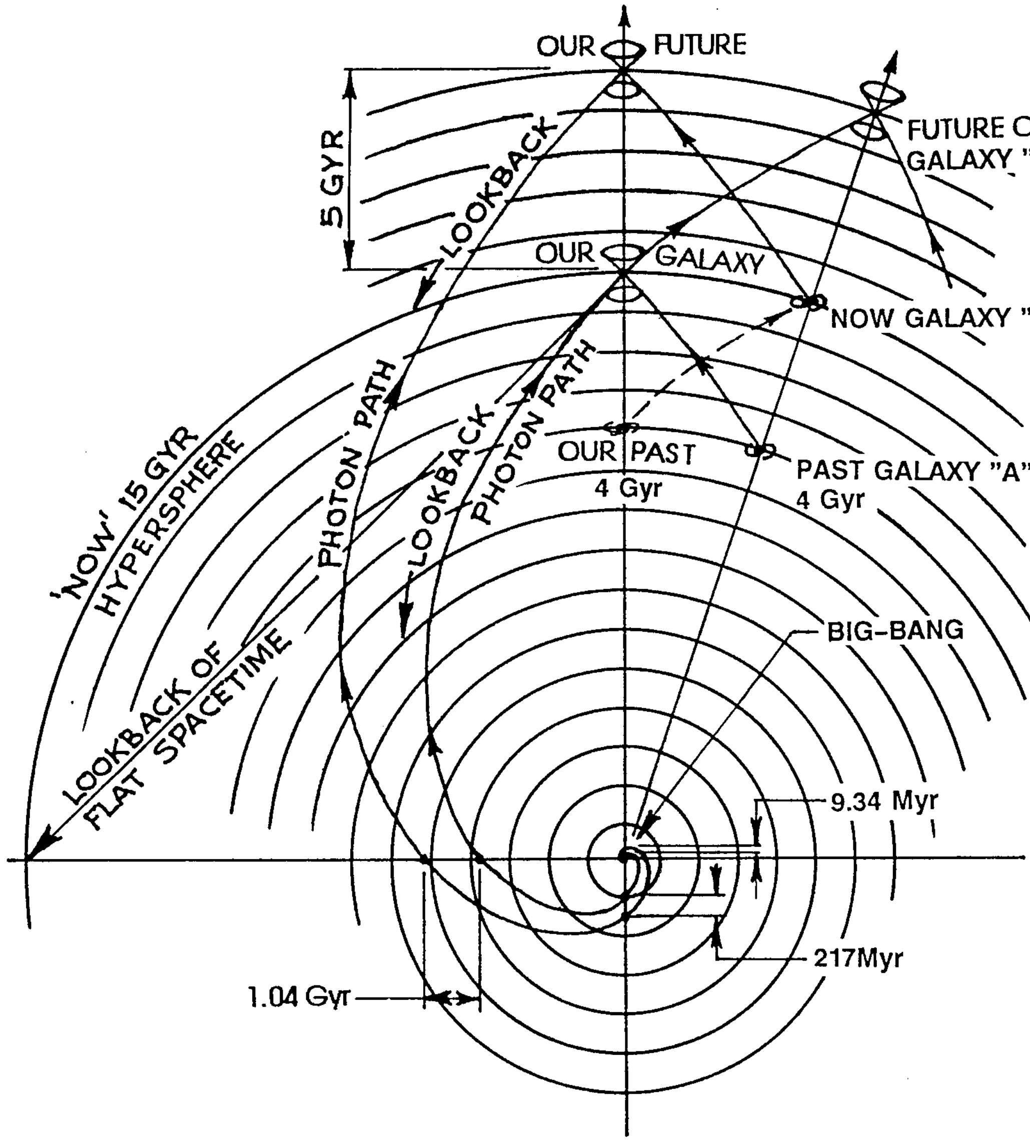

Figure 5. Observed distortion in object length and time flow.

explaining it by invoking a big bang model with standard flat space-time light cones and a system expansion rate (based on use of a local clock) corresponding to

$$c' = c \exp(\phi) \quad (14)$$

or a piece-wise linear approximation of Eq. (14). This would suggest a very high initial expansion rate for the system near $T \approx 0$, and a normal rate corresponding to c close to the location of the observer. Such features are reminiscent of those found in the "inflationary universe" model.[13] Thus the expanding hyperspherical space-time model described here offers support for the concept of an "inflationary beginning" by presenting it as a locally perceived artifact of its topology.

A related comment concerns the "horizon problem",[14] one of the parent sources of the "inflationary universe." If one accepts the notion of global spherical curvature presented in this paper, it can be seen that the "horizon problem" for such a curved topology does not exist, since all observed lines of sight terminate at the big bang origin. From this viewpoint, conventional "standard model" cosmology appears then to require an initial inflated state of unknown origin merely because its use of standard flat space-time light cones (and straight-line photon trajectories) fails to properly accommodate the topology of its intended spherically expanding target. Viewed within the context of the space-time viewpoint presented here, the use of an

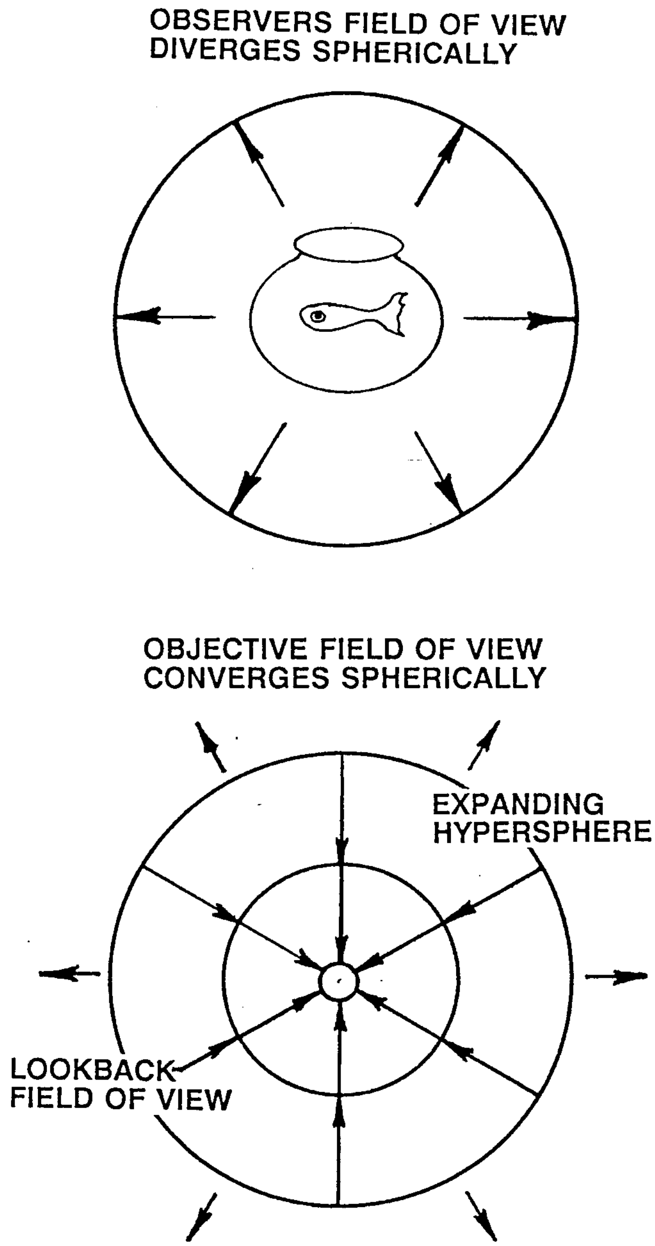

Figure 6. Mercator-projection-like distortion in observed field of view.

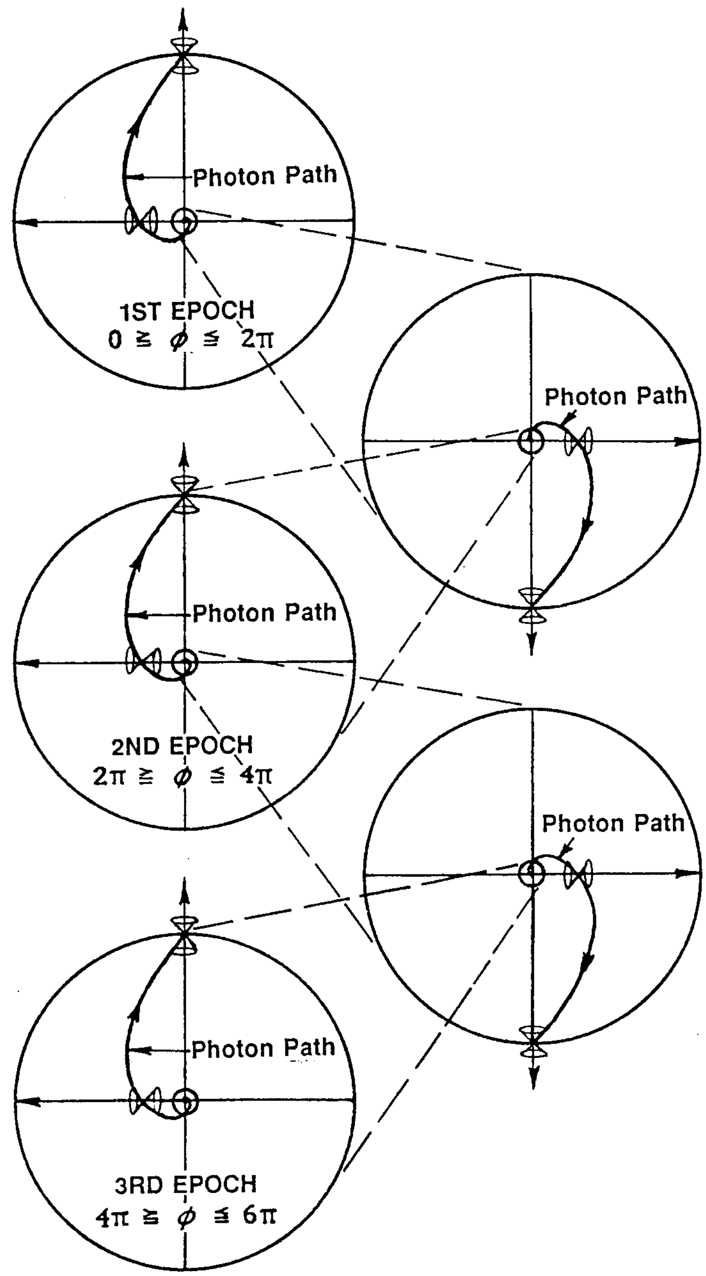

Figure 7. Recurring "epochs" of observability in observed field of view.

inflation concept to repair the standard model "horizon problem" can be interpreted as a vehicle for tacitly introducing spatial curvature. However, as indicated in Eq. (14), the use of an expanding hyperspherical space-time model allows one toachieve the same goal in a more direct and natural manner.

4.2 Cosmic Background Radiation

The model employed here is a topological derivative of a global model that was developed to describe Friedmann–Lemaître expansion systems. It has been specifically designed to accommodate spherical systems with initial conditions relating to a big bang singularity and local light cone attributes matched to the systems geometry. One might therefore expect a relatively good hand-in-glove fit between predictions of the model and big bang residual background radiation observations. Several points will now be reviewed to illustrate the degree to which this is true.

First of all, some working parameter values will be defined to support discussion in terms of results that can be understood quantitatively. These include the current age of the universe T, and bounds on the range of $t = t_p$, the time locus of the observable big bang explosion event. The value of T is much debated, but most reported data generally fix the current age of the universe in the range[15]

$$10^{10} \leq T \leq 2 \times 10^{10} \text{ yr}. \tag{15}$$

A working value of $T \approx 1.5 \times 10^{10}$ yr (or 5×10^{17} s) will therefore be assumed. An upper bound on the value of t_p is frequently quoted as 3×10^5 yr,[16–18] the time at which the big bang radiation level has cooled sufficiently to have allowed decoupling of matter. The fact that the observed background radiation exhibits a scattered "blackbody" spectral signature also supports the viewpoint that the most probable event locus for t_p is in the vicinity of this upper bound. The elapsed time certainly exceeds the Planck time $t = T^*$. Consequently, the time locus of the observable big bang event will be taken as

$$T^* < t_p \leq 3 \times 10^5 \text{ yr}. \tag{16}$$

The foregoing working parameter values will now be utilized within the context of what is inferred by the nature of the model and the analysis results developed in the prior sections. The first point to be made concerns the fact that it correctly predicts observability of background radiation relating to the big bang explosion: A big bang explosion with observable event locus at $t \approx t_p$ will be continuously observable from all locations of the now-hypersphere for $t > t_p$ and along all possible lines of sight. This is demonstrated as follows:

(1) By design, all photon trajectories in the model must intersect the origin point of the expansion system as well as hyperspherical surface of finite radius enveloping the origin point.
(2) By virtue of geometric symmetry, the photon trajectory locus given in the analysis section is valid for all possible choices of observer location on the now-hypersphere and all choices regarding line of sight.
(3) Since photon trajectory intersection with the explosion event locus is independent of t, observability of that event is continuous.

A second inference of the model regards the relationship of the background radiation to other objects located closer to an observer on the now-hypersphere. Since the model manifests a "many-to-one" mapping of the same regions across a sequence of time interval epochs, this suggests that observation of the background radiation could potentially be correlated with information about objects in the same regions appearing in later epochs nearer to the observer. The utility of the background radiation information as a correlation reference vehicle, however, would depend upon the extent to which the observable event locus of the big bang explosion is crisply defined, and the relative angle ϕ between it and the observer. As indicated in the cosmological observability discussion, Sec. 4.1, the spiraling nature of the photon path causes the actual time span of an epoch to be very large near the vicinity of the observer and very short for those at greater distances. Thus facility to correlate is very coarse for events located relatively near the observer and rather fine for events in the very distant past. If one assumes that the most probable choice for t_p is the upper bound given in (16), this would suggest that the observable big bang explosion event locus resides at $\phi \approx 3.5\pi$, the far side of the epoch just beyond the initial one which includes the locus of the observer at $\phi = 0$. This implies that there is at most a pair of sightings on the same region associated with a selected line of sight and that the facility to correlate is very limited. Moreover, since the epochs beyond the first two are hidden behind the observable locus of the explosion event, the region of fine resolution would not be observable.

A final point of discussion on the topic of background radiation concerns the impact of recession velocity on an observer's view of the background radiation from the now-hypersphere at $t = T$. The model suggests that the observed background radiation at $t = T$ will be redshifted by a factor

$$z_p = \phi_p = z_T - z_{t_p} = \ln(T/t_p) \tag{17}$$

which in turn yields

$$138 > z_p \geq 11. \tag{18}$$

The lower bound in Eq. (18) corresponds to the upper bound given for t_p and therefore it can be regarded as the most probable value. The expressions in (17) and (18) imply that the early universe was hotter at the locus of $t = t_p$ by a factor of z_p. This follows from the fact that such a redshift has the impact of lowering the observed background radiation temperature by a factor z.

4.3 Quasar Observations

Quasars are generally characterized as very distant cosmological objects with high redshift and very high luminosity. The analytical results presented here provide a natural basis for explaining the high redshift and the extreme distance attributes of such objects. On the other hand, the "lens effect" enlargement distortion inferred by the results has the impact of reducing luminosity and offers no basis for explaining luminosity levels that match those of entire galaxies.

Reported distance and age estimates for quasars in the published literature appear to be principally derived using analytical models based upon the use of Hubble's rule for distance and recession velocity in flat space-time. Such a model typically assumes a constant rate of expansion such that

$$T = 1/H, \tag{19}$$

where H is the Hubble constant. The model also assumes a special relativity form Doppler effect equation to approximately correct for the impact of time dilation and length contraction on the observed redshift

$$z = \frac{\Delta\lambda}{\lambda} = \left[\frac{1 + v/c}{1 - v/c}\right]^{1/2} - 1, \tag{20}$$

where $\Delta\lambda$ is the observed wavelength shift of a wave with local wavelength λ, and v is the time dilation corrected recession velocity. The resulting expression for estimated distance, then, is

$$d' = \frac{v}{H} = cT\left[\frac{(z+1)^2 - 1}{(z+1)^2 + 1}\right]. \tag{21}$$

If one compares the distance expression in Eq. (21) with the derived formula in Eq. (9), it can be seen that both have the basic form

$$d = cTf(z), \tag{22}$$

where $f(z)$ is a function of z. The $f(z)$ term in Eq. (21), however, does not appear to be a commonly known approximation of the $[1 - \exp(-z)]$ term in Eq. (9). Nevertheless, as shown in Table II, a quantitative comparison of equations (9) and (21) shows the two distance expressions are remarkably similar to each other. As can be seen in the table, the two expressions are matched at the limiting values of $z = 0$ and $z \to \infty$ and have a worst case separation of about 8% at $z = \pi$. It must also be noted, on the other hand, that the distance value for Eq. (9) converges more quickly to cT than that of Eq. (21) as the value of z grows in magnitude beyond $z = \pi$. The close agreement between the two distance expressions, otherwise, clearly begs for explanation.

Table II: Comparison of Distance Formulas (9) and (21)

Observed Redshift $z = \phi$	*Exact Distance* d	*Special Relativity Approx. Distance* d'
0	0.0	0.0
1	0.63cT	0.60cT
$\pi/2$	0.79cT	0.74cT
π	0.96cT	0.89cT
2π	0.998cT	0.963cT
3π	0.9999cT	0.982cT
4π	0.999996cT	0.987cT
$\to\infty$	1.00cT	1.00cT

The explanation for the close agreement between Eqs. (9) and (21) is as follows. First of all, it should be noted that (as a result of the cT term) both expressions are based on use of the Hubble rule regarding distance and recession velocity. Within this context, it can be seen that Eq. (21) can be derived by simply imposing the special relativity Doppler correction for velocity defined in Eq. (20) on the equation for observed distance in Eq. (12). When the observed velocity term $c\Phi = cz$ in Eq. (12) associated with the observed redshift is Doppler corrected per Eq. (20) to become equal to v, the distance D in Eq. (12) is thereby transformed to become equal to the distance expression for d' given by Eq. (21). Another reason for doing such a derivation is that the expression in Eq. (21) can then legitimately be associated with a space curvature mechanism which supports redshift factor values above the limit $z = 1$.

A second point regards how the analysis given in this paper relates conceptually to the theory of relativity itself. It should not come as any surprise to have it suggested that detailed information on photon path locus between observer and an object is equivalent to knowing information about observed relativistic space-time transformations that affect the observer's view of the object. Indeed, relativity also expresses itself basically in terms of the same modifications to length and time flow which were considered in the analysis section to describe distortion. Use of a Penrose–Hawking global model baseline for defining the hyperspherical space-time model allows one to associate the photon trajectory locus results obtained thereby with observed relativistic space-time transformations in Friedmann–Lemaître expansion systems. The photon trajectory locus results may therefore be regarded as general relativity consistent. Thus one would expect close agreement between those results and others based on special relativity.

On the other hand, the special relativity solution found in Eq. (21) is not tailor-made to suit the geometric and dynamic requirements of its intended target. The space-time topology imposed by general relativity is necessarily spherical, and photon trajectories follow a spiraling path locus. Thus the expression in (21) provides only an approximation of an exact solution. This occurs because of a gauge mismatch between its flat space-time basis and the spherical topology of the target. The extent to which Eq. (21) fails can be evaluated by comparing it with Eq. (9). As indicated by the results in Table II, the model basis for Eq. (21) does not assure correct local time cone alignment across all event horizon boundaries. If it did, its solution would be exactly equivalent to that given in Eq. (9).

Although the special relativity-based expression in Eq. (21) gives remarkably good distance estimates in comparison to an exact solution, its estimates of quasar age, however, can contain errors of significant magnitude especially at observed quasar redshift values greater than $z = \pi/2$. A survey of recently published results regarding the quasar age suggests an estimated age viewpoint that significantly exceeds that suggested by the analysis results given in this paper. For example, one report,[19] indicates that quasars with redshift $z \approx 3$ have an age $\leq 20\%$ the current age of the universe (i.e., $0.20T$). This limit is almost twice as high as that predicted by the special relativity-based solution for distance given in Eq. (21). The formula in (9), however, suggests that the age for such a quasar is $0.04T$. Another paper[20] indicates that quasar formation is believed to take a very long time because the time to grow a massive central black hole is long (estimated to be in the range 0.3 – 1.0 billion years). The model used in this paper indicates that for quasars with $z \approx 5$, approximately at the limit of current observations, the age would be $0.0067T$, or about 100 million years. Thus the results given here, based on the notion of global spatial curvature, call into question the validity of using special relativity flat space-time based models to generate quasar age estimates.

5. CONCLUSIONS

At first sight, the expanding hyperspherical space-time model presented here appears to have little in common with the consensus accepted flat space-time viewpoint. However, in its favor, one should take note of the following positive points:

(1) It provides an intuitively appealing representation of the big bang event as an "expansion of space" process.
(2) It has a strong analytical basis rooted in global models for black holes and Friedmann-Lemaître expansion systems.
(3) A form of the Hubble rule can be directly derived from it which supports observed redshift factor above $z = 1$.
(4) It provides a natural topological basis for avoidance of the "horizon problem" encountered in flat space-time.
(5) It supports the notion of an "inflationary beginning" as a locally perceived artifact of its topology.
(6) There is natural support for continuous observability of the cosmic background radiation.
(7) It yields quasar distance results that are in close agreement with those of special relativity-based models.
(8) It correctly models locally perceived time asymmetry.
(9) Its topological basis greatly reduces analytical complexity.
(10) Its local framework asymptotically approaches flat space-time as photon path length becomes short.

On the negative side, it can be noted that the concept of "lens effect" distortion appears to be severely at odds with the "what you see is what you get" implication of flat space-time. But the results given here indicate that a local framework with flat space-time and an exponentially defined inflation process can be exactly equivalent to the local framework of an expanding hyperspherical space-time model.

Thus the unusual space-time model presented here can have a sensible connection with commonly accepted flat space-time if one includes an appropriate inflation concept. In this regard, one might choose to view the new space-time model as a topological representation of a special kind of (locally perceived) "inflationary universe." Conceptually, however, it would probably be more beneficial to simply view inflation as a tacit means for introducing curvature in a flat space-time setting.

The positive nature of the results presented here provide a promising initial snapshot view of the new hyperspherical space-time model and some of its implications. Clearly, more exploration is required to validate and extend these initial results.

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Résumé

Nous dérivons des expressions analytiques pour caractériser les propriétés de la trajectoire du photon dans une hypersphère de Friedmann-Lemaître en expansion. Les résultats obtenus impliquent l'existence de distorsions qui peuvent modifier de façon importante l'image locale d'objets distants en termes de position, de distance, de grandeur et de taux d'écoulement du temps dans un tel référentiel. Nous examinons les effets de ces distorsions sur les observations cosmologiques, les rayons cosmiques, et les images de quasars. Nous montrons que la nature du champ de vision observé prédit analytiquement est non seulement celui qui satisfait les exigences de distance et de vitesse de récession de la loi bien connue de Hubble, mais aussi prédit les décalages vers le rouge au-delà de $z = 1$*. Les conséquences de cette trajectoire de photons sur la température de la radiation du fond cosmique suggèrent toutefois un besoin de correction pour* $z \geq 11$*. L'expression dérivée pour la distorsion peut être aussi considérée comme équivalente aux solutions obtenues dans le cadre de la relativité générale, en ce qui concerne les transformations de l'espace-temps dans une hypersphère en expansion. Nous comparons nos résultats avec ceux dérivés de la relativité restreinte pour l'évaluation de la distance d'un quasar. Les deux solutions prédisent de valeurs très voisines* ($0 \leq$ *erreur* $< 8\%$)*. Bien que proche de celle du décalage vers le rouge* $z < \pi/2$*, l'expression prédit l'âge, diverge rapidement pour* $z > \pi/2$*.*

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