The Arrow of Time in an Expanding 3-Sphere

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Abstract

An adapted version of the semiclassical 3-sphere tunneling model is applied to a quantum cosmology scenario that features nucleation at the Planck energy density limit and a radiation-dominated phase that slowly transforms into a matter-dominated phase. In the classically allowed range, the closed spherical geometry of the model is constrained topologically to define a "late-blooming" Einstein–de Sitter universe with locally perceived flat space-time and expansion-synchronized time-flow. The predicted time-flow attributes of this model are then used as a basis to evaluate how well-observed values of age, Hubble flow rate, and deviation of Hubble law as a function of brightness compare with those of the model. It is shown that, although there is strong agreement between observed data and the predictions of the model to support the view that time-flow is expansion synchronized, improved resolution on Hubble flow-rate measurement is required for a definitive conclusion. A process level connection between 3-sphere spatial expansion and locally observed CBR plane-wave spreading is made in terms of an observed expansion-caused transformation of space-time. This gives evidence to illustrate how expansion of the 3-sphere can serve as the causal basis for the "arrow of time" dynamic seen locally in the propagation of electromagnetic fields.

Keywords: arrow of time, expanding 3-sphere, Friedmann–Lemaître model, inflation scenario, mini-superspace, quantum clock, quantum cosmology, semiclassical model, semi-eternal inflation, temporal flow, time, time dynamic, tunneling wave-function, Wheeler–De Witt equation

1. INTRODUCTION

In quantum cosmology, the strong interest in defining a set of initial conditions for the conception state of the universe leads one rather naturally to consider big-bang system expansion as a preferred basis for a quantum clock.[1–3] Alternative clocks such as (i) a scalar field associated with vacuum fluctuation and matter distribution, and (ii) classical mechanisms based on thermal entropy, gravity, decay mechanisms, and radiation have also been proposed. Except for entropy, however, these are seldom viewed as primary mechanisms.[3] Use of a particular degree of freedom as a "master clock" in a quantum-state prescription of the universe suggests that if that clock were to halt, the quantum state would freeze statically (or undergo alteration) such that all dynamics observable at the classical level would cease. Thus, if a selected clock mechanism is portrayed as the "master arrow of time," one must accept the challenge of demonstrating a strong process-level association between a quantum-based clock dynamic and observed classical time-flow. This paper will attempt to pursue such a challenge by offering some results that argue in favor of viewing the expansion dynamic of a 3-sphere based on the Friedmann–Lemaître model as the wellspring of its observed time dynamic. The scope of the presentation given here is restricted to the use of a semiclassical quantum tunneling model for a closed expanding 3-sphere and classical extensions peculiar to the topology of such a space-time framework. Analytical results are developed for a standard model like scenario that features

(i) nucleation exactly at the Planck limit,
(ii) a "late-blooming" form of an Einstein–de Sitter universe modeled as a 3-sphere expanding at a constant rate, and
(iii) a homogeneously modeled form of "semi-eternal" inflation.

These results strongly focus on the nature of what follows nucleation as opposed to the nucleation process itself. As currently defined in published literature, the quantum cosmology tunneling model developed by Vilenkin[4–9] is restricted to

i) "bounded-from-above" nucleation potentials that are much smaller than the Planck limit on energy density, and
(ii) an inflation scenario where the lifetime of the false vacuum, although constrained to be much greater than $1/H$, is implied to end rather abruptly when the vacuum energy thermalizes completely (i.e., $V(\phi) \to 0$) at an early stage in the classically allowed range.

An adaptation of the tunneling model must therefore be made to circumvent these restrictions, but without introducing attributes that would invalidate the use of the WKB approximation. One might also consider use of the Hartle–Hawking proposal[10–13] for a path-integral–based definition of a quantum wave-function, but this requires a nucleation potential well above the Plank limit. It is not immediately clear how such a wave-function can be modified to validly support nucleation exactly at the Planck limit within the context of the WKB approximation.

Semi-eternal inflation, as currently defined,[14–17] regards a scenario in which vacuum fluctuations thermalize stochastically and inhomogeneously. Large, distinctly separate, mutually unobservable regions of the universe undergo thermalization in a process that never completely ends. The observable universe as seen locally is then defined on a flat space-time region that has completed thermalization, inflated via an abrupt symmetry-breaking process[18] at some finite time in the past, and is out of touch with the other regions. Unfortunately, such a scenario by its nature lends itself neither to homogeneous modeling nor to direct observational testing within the context of local analysis. Global modeling methods, however, potentially provide the means to circumvent the natural limitations of local models by taking a broader view of the system. When a 3-sphere model is specifically constrained to a topological set that requires time-flow to be a linear function of system expansion globally, it can innately support semi-eternal inflation within the context of its global expansion dynamic. The analysis presented here defines such a model in terms of a constraint set that forces it to be an approximation of a late-blooming Einstein–de Sitter universe. The particular scenario employed is standard-model–like with a radiation-dominated phase that transforms with slow exponential decay into a matter-dominated phase with mass-density parameter $\Omega \to 1$. This model is constrained so that

(i) the matter-dominated expansion phase begins at decoupling time t_d when the matter to radiation energy density ratio $\rho_m/\rho_r \approx 1$;
(ii) flat space-time is seen locally as $\Omega \to 1$ after decoupling time; and
(iii) the global rate of expansion is approximately constant.

In the classically allowed range, the topology of the expanding (Friedmann–Lemaître model based) 3-sphere can be exploited by use of a global exterior perspective Schwarzschild-like space-time framework to enable simplified representation of light cone properties.[19–21] This vehicle, in turn, provides a convenient way to evaluate photon trajectory impacts associated with the expansion on local observations. Such a technique will be employed to generate predicted values of age, Hubble flow rate, and deviation from the Hubble law as a function of observed object brightness. These predicted values are compared with measured values to determine the extent to which time-flow is synchronized with 3-sphere expansion. Observed data of several kinds will be invoked to test the validity of using system-expansion scale as a clock for measuring actual temporal flow macroscopically. Included are

(i) star cluster age as indicated by stellar main sequence turnoff,
(ii) Hubble flow rate for a large variety of measurement methods, and
(iii) deviation from the Hubble law as a function of object brightness in field galaxy and radio galaxy surveys.

Although such data cannot be used to establish the existence of a causal relationship between time-flow and expansion, negative results would indicate its absence. In order to mechanistically associate the expansion scale factor of the 3-sphere with classically observed temporal flow, the impact of observed relativistic space-time transformations that occur in connection with volumetric spreading of CBR electromagnetic energy will be examined. These will be considered as a basis for establishing how spatial spreading due to expansion can be seen as the cause of the plane-wave propagation of electromagnetic energy per Maxwell's equations in local classical observations.

2. NUCLEATION/EXPANSION SCENARIO

The singular initial state commonly encountered in most classical cosmology model solutions of the general relativity field (GRF) equations is often viewed as an object that must be tamed with "cosmic censorship" or circumvented with quantum mechanics (QM). Analytically, the potential hazards posed by such a singularity are related to a lack of physical realizability and a breakdown in deterministic predictability due to an ill-defined initial state.[22] One reaction to such an obstacle is to proceed as if "the boundary condition of the universe is that it has no boundary."[23] This viewpoint is more formally defined in the Hartle–Hawking proposal[10] that "the path integral for quantum gravity should be taken over all compact Euclidian metrics without boundary." The "tunneling from nothing" proposal by Vilenkin represents another QM-based alternative, and this is the model that is employed here. Regardless of which model one employs, however, there must be a mutual coherence between the quantum-mechanical model and classical representation with regard to what is known both analytically and observationally. One of the features of the tunneling model is that it predicts an initial state of the universe that nucleates with maximum vacuum energy density. This corresponds rather well to what is qualitatively implied by the classical singularity mentioned above. In terms of the simplest quantitative interpretation, this suggests that the singularity state (with energy density $\rho \to \infty$ as $t \to 0$) corresponds to a QM state with maximum energy density at the minimum time state, i.e., $\lambda(\rho) \to L^*$ (Planck length) as $t \to T^*$ (Planck time), where $\lambda(\rho)$ is the energy density wavelength. In this context, the classical singularity mentioned above, if it honors such a constraint, can begin to correspond to a state that is both physically realizable and well defined. If the energy density state at $t \to T^*$ is interpreted as being strictly associated with matter distribution, however, coherency can be at risk. The classical nucleation/expansion scenario implied by standard model cosmology when coupled to inflation typically assumes

(i) abrupt initial quasi-exponential expansion driven by a potential $V(\mathrm{vac})$ attributed to false vacuum fluctuation,[18]
(ii) thermalization when the potential $V(\mathrm{vac}) \approx 0$ to commence a radiation-dominated expansion phase, and
(iii) transition to a matter-dominated expansion phase at decoupling time t_d when the energy density of decoupled matter is equivalent to that of the radiation.

Since ϕ is a scalar field associated with matter distribution, it is also commonly assumed that the total system potential energy $V(a, \phi)$ is at a maximum and $\phi \approx 0$ when scale factor a is small. Such a viewpoint is consistent with the implications of the QM-based tunneling model. To conserve energy at the start of the radiation-dominated phase, it is therefore necessary that $V(a, \phi) = V(\mathrm{vac})$. The relatively late advent of the matter-dominated expansion phase, however, suggests that the rate of change of $V(a, \phi)$ remains small until one nears the decoupling time. Within this context, one can define an initial potential $V(a, \phi)$ at the Planck limit that will not violate the requirements of the WKB approximation if the rate of change of $V(a, \phi)$ is slow enough. Viewed as a coherency constraint on the singularity mentioned above, this suggests that thermalization of the potential $V(\mathrm{vac})$ corresponds to a potential $V(a, \phi)$, which manifests itself principally as an electromagnetic field during the radiation-dominated expansion phase.

A remaining issue concerns the exact form of $V(a, \phi)$ to be used. It appears that the simplest QM model expression that can be used to approximate the slow decay of $V(a, \phi)$ during the radiation-dominated phase and its abrupt demise at the start of the matter-dominated phase is an exponential form with a parameter-controlled negative exponent. This facilitates an exact coherency match between QM and classical models for a system state at $t = 0$, $t = t_d$, and $t \to \infty$, not to mention simplified analysis.

Within the context of expansion-synchronized time-flow, $V(a, \phi)$ can be defined simply as a function of the expansion scale factor a. The wave equation for the tunneling model can then be developed for three separate phases of expansion with $V(a, \phi)$ potentials:

Euclidian region phase:

$$V_1(a,\phi) = 1, \qquad 0 \le a \le 1. \tag{1}$$

Radiation-dominated phase:

$$V_2(a,\phi) = \exp[-K(a-1)] \ge \frac{1}{2}, \qquad 1 < a \le a(t_d). \tag{2}$$

Matter-dominated phase:

$$V_3(a,\phi) = \exp[-K(a-1)] \le \frac{1}{2}, \qquad a(t_d) \le a < +\infty. \tag{3}$$

The unity state for $V_1(a, \phi)$ corresponds to a Planck level energy state (with cgs wavelength L^*), which is constant until $a = 1$ (scale factor at cgs time T^* assuming speed-of-light expansion). At decoupling time t_d, the boundary between the radiation-dominated phase and the matter-dominated phase, the potential $V(a, \phi) = 1/2$, which implies a balance between radiation field and matter field energy density. Assuming speed-of-light expansion and a decoupling time of 2×10^5 years (or $10^{55}T^*$), the corresponding scale factor value is 10^{55}. This, in turn, implies that

$$\exp[-K(a(t_d)-1)] = \frac{1}{2}, \; a(t_d) \approx 10^{55}, \text{ and} \tag{4}$$

$$K \times 10^{55} \approx \ln\frac{1}{2} \text{ or } K \approx 10^{-55}. \tag{5}$$

The relationship between potential and matter-based energy density during the latter two phases, as implied by (2) and (3), can be defined as

$$\frac{m^2\phi^2}{2} = 1 - V(a,\phi) = 1 - \exp[-K(a-1)], \; 1 < a < +\infty, \tag{6}$$

where m is mass density. The state of the term m during the matter-dominated expansion phase is assumed to be at the critical level required to support locally observed flat space-time. It will be necessary to demonstrate that this is consistent with an initial energy density state at the Planck limit. Flat space-time would also imply a closed system that ultimately collapses to a unique singularity if one ignores the impact of a protracted radiation-dominated expansion phase and the possibility of an alternative terminal singularity form. Although the scenario considered here concerns an inflationary model that is semi-eternal, this does not preclude an ultimate collapse of matter locally into a distributed set of black-hole objects. The occurrence of such an event depends upon local inhomogeneities that on average are indistinguishable in the homogeneous/isotropic distribution of the scalar field ϕ. The expectation for a final crunch state implied by classical closed-system solutions of the GRF equations is therefore potentially realizable within this context.

3. ADAPTED TUNNELING MODEL

The fundamental basis for the tunneling model is given by the Wheeler–De Witt equation[24–26] for a wave-function Ψ defined on a superspace manifold of all possible 3-space metrics and matter fields:

$$H\Psi = 0, \tag{7}$$

where H is the momenta-mapped Hamiltonian constraint implied by the GRF equations for the manifold. In a closed space-time framework that is homogeneous and isotropic, this manifold can be reduced by virtue of geometric symmetries to a form with two degrees of freedom: expansion scale factor a and matter distribution scalar field ϕ. In the case of the Vilenkin "tunneling from nothing" model,[8] these degrees of freedom are defined on a closed spherical manifold with Robertson–Walker metric

$$ds^2 = \sigma^2[dt^2 - a^2(t)d\Omega_3^2], \tag{8}$$

where $\sigma^2 = 2G/3\pi$ is a normalizing factor, G is the gravitational constant, $d\Omega_3$ is the metric on a unit 3-sphere, $\hbar = c = 1$, and

$$R = 6a^{-2}(1 + \dot{a}^2 + a\ddot{a}) \tag{9}$$

defines the scalar curvature of the structure. Use of the Lagrangian

$$\mathcal{L} = \ell_p^{-2}R + \frac{1}{2}(\mu\phi)^2 - V(\phi), \tag{10}$$

to characterize interactions between gravity and the scalar field ϕ, where $\ell_p = (16\pi G)^{1/2}$ is Planck length, defines a model with action

$$S = \int d^4x(-g)^{1/2}[\mathcal{L}], \tag{11}$$

where g is the determinant of the metric tensor. The partial derivatives of the Lagrangian with respect to a and ϕ can then be taken to define canonical momenta in a Hamiltonian for the spherical manifold. Replacement of the momenta with Schrödinger partial derivative operators, together with the use of the relationships

$$\phi = \left(\frac{4\pi G}{3}\right)^{1/2}\phi \text{ and } V = \left(\frac{4G}{3}\right)^2 V, \tag{12}$$

yields a wave equation that is specific to mini-superspace in a 3-sphere:[8]

$$\left[\frac{\partial^2}{\partial a^2} + \frac{p}{a}\frac{\partial}{\partial a} - a^{-2}\frac{\partial^2}{\partial \phi^2} - U(a,\phi)\right]\Psi = 0, \tag{13}$$

where

$$U(a,\phi)^2 = a^2[1 - a^2V(\phi)],\ 0 < a < \infty,\ -\infty < \phi < +\infty, \tag{14}$$

$$V(\phi) = \rho_v - \frac{m^2\phi^2}{2}. \tag{15}$$

The parameter p in (13) represents the factor ordering ambiguity of variables a and $\partial/\partial a$, and ρ_v in (15) is vacuum energy density. In the present exercise, this model will be adapted to accommodate the scenario defined in the prior section. The 3-phase standard-model–like scenario considered here requires the scalar field ϕ to be a function of scale factor a. This permits tailoring of the potential function in (15) and the Lagrangian in (10) to fit the particular needs of each phase. In the first phase, the potential $V(a, \phi)$ remains fixed at the Planckian limit until $U = 0$, the border between the Euclidian and Lorentzian regions of the model. During this phase, the Lagrangian $\mathcal{L}$ and potential V take the form

$$\begin{gathered}\mathcal{L}_1 = \ell_p^{-2}R - V_1(a,\phi),\ V_1(a,\phi) = 1,\\ 0 \le a \le 1,\ U(a,\phi) > 0.\end{gathered} \tag{16}$$

The constraints in (16) lead to a de Sitter space solution[8,9] for a simplified form of (13):

$$\Psi_1 = (1-a)^{1/4}\exp([(1-a)^{3/2} - 1]/3),\ U > 0,\ 0 \le a < 1 \tag{17}$$

in the Euclidean under-barrier region. As indicated by (14), at the boundary, scale factor $a = 1$, which indicates that it is at the Planck limit. It should be noted that this corresponds logically to the suggestion given in the previous section for a simple interpretation of the classical singularity, i.e., that it map to a quantum state with $\lambda(\rho) \to L^*$ as $t \to T^*$ in cgs units.

During the radiation-dominated phase in the classically allowed range, the Lagrangian $\mathcal{L}$ and potential V are defined by

$$\begin{gathered}\mathcal{L}_2 = \ell_p^{-2}R - V_2(a,\phi),\ V_2(a,\phi) = \exp[-K(a-1)],\\ 1 < a < a(t_d).\end{gathered} \tag{18}$$

Near the boundary where $U \approx 0$ and $\exp[-K(a-1)] \approx 1$, the WKB method may be applied to obtain a wave equation solution for the start of the radiation-dominated phase, which is a simple plane wave:[8]

$$\Psi_2 = \exp ik(\alpha \pm \phi) \approx \exp ik(a/2.73),\ U \approx 0,\ a \approx 1, \quad (19)$$

where $\alpha = \ln a$. Thus, it can be seen that up to this point the solution forms for the scenario considered here are essentially similar to those encountered when nucleation is restricted to a level well below the Planck limit. As one proceeds further into the classically allowed range, however, the solution forms diverge from this baseline to manifest a set of attributes that correspond more closely to what might be expected macroscopically.

Away from the vicinity of the boundary where $a \gg 1$, the potential term in (13), $U(a, \phi)$, can be approximated as $a^4 \exp[-K(a-1)]$, and partial derivative terms in (13) other than the first term become negligible. Application of the WKB approximation $\Psi \approx \exp(iS)$ then yields a solution

$$\Psi_2 = \exp i([a^3/3]\exp[-K(a-1)/2]),$$
$$U < 0,\ 1 << a << \frac{6}{K}, \quad (20)$$

which suggests volumetric spreading of energy since the wavelength of Ψ_2 is proportional to a^3. This is not unexpected since there is insufficient matter present during the radiation-dominated phase to support locally observed flat space-time. It can also be seen that the wavelength of Ψ_2 in (20) is proportional to the term $\exp[-K(a-1)/2]$. This illustrates the impact of scalar field formation on the wavelength of the wave-function during the course of the radiation-dominated phase. At decoupling time, radiation-dominated expansion ends, and a matter-dominated expansion phase commences with Lagrangian $\mathcal{L}$ and potential V as defined by

$$\mathcal{L}_3 = \ell_p^{-2} R + \frac{1}{2}(\partial_\mu^2 \phi) - V_3(a,\phi),$$
$$V_3(a,\phi) = \exp[-K(a-1)],\ a > a(t_d). \quad (21)$$

The nonlinear form of V gives rise to a solution for Ψ_3 in this region that differs markedly with that shown in (20), even though V_2 and V_3 are similarly defined except for the range of scale factor a. Invocation of the WKB approximation with the constraint set in (21) yields a wave equation solution

$$\Psi_3 = \exp i([2a^2/K]\exp[-K(a-1)/2]),$$
$$U < 0,\ \frac{4}{K} << a < +\infty. \quad (22)$$

It is worth noting that this solution form implies a spreading of energy during the matter-dominated expansion phase that follows an inverse-square relationship. Such an attribute is normally associated with the existence of flat space-time in a classical setting. Its occurrence during the matter-dominated phase appears to be the result of having chosen a scenario in which $V(a, \phi)$ decays sufficiently at large values of the scale factor so that the magnitude of $m^2\phi^2/2$ per (6) becomes dominant in comparison to other factors. This dominance is not sensitive to the magnitude of $V(a, \phi)$, however, and thus imposes no specific requirement on mass density except as implied by the closed spherical geometry of the model. At the boundary between the radiation-dominated phase and the matter-dominated phase, the wave-function solution takes a form that is a linear combination of both solutions:

$$\Psi = \exp[(i/2)([4a/K - Ka^3/6]\exp[-K(a-1)/2])],$$
$$U < 0,\ a \approx \frac{5}{K}. \quad (23)$$

Since the adapted model used here makes successful use of the WKB approximation, it can be seen that the requirement that it nucleate with maximum energy density[8] remains valid. When that nucleation occurs at the Planck limit, the adapted tunneling model also maintains coherency with an interpretation of a classical singular initial state that corresponds to a maximum energy density at the minimum time state. This establishes the basis for a well-defined set of initial conditions to support classical analysis. The topological analysis presented in the next section will make use of this baseline to generate predicted results regarding the relationship between 3-sphere expansion and observed time-flow.

4. TOPOLOGICAL EVALUATION

As indicated in the introduction, the object of this paper is to explore the degree to which locally observed temporal flow in an expanding 3-sphere can be defined as a linear function of the expansion scale factor a. The point of focus for this exploration strongly concerns observed age and temporal flow attributes of very distant objects. The class of analytical solutions that are hypothetically tested within this context correspond topologically then to a set of 3-spheres that are expanding essentially at a constant rate. A number of additional constraints must be invoked to reduce this set to a subset that is cosmologically and physically relevant. To be compliant with the scenario defined in the prior sections and locally observed physical attributes, the expanding set of 3-spheres is constrained to a subset with

(i) nucleation at a Planckian energy density level in a closed system that conserves energy,
(ii) an expansion process governed by the GRF equations featuring a radiation-dominated expansion phase that gradually transits to a matter-dominated phase,

(iii) locally observed flat space-time during the matter-dominated phase,
(iv) locally observed redshift for distant objects that is consistent with the Hubble law, and
(v) conservation of locally observed luminal flux.

An additional constraint implied by the flat space-time requirement within the context of the GRF equation solution for a unit sphere is that matter energy density $\rho_m = \rho_c$, the critical level associated with locally observed zero curvature. This, in turn, requires that the matter to radiation energy density ratio $\rho_m/\rho_r \approx 10^4$ at time t = now during the matter-dominated phase. If the above-defined constraints are imposed without requiring that time-flow be explicitly dependent on 3-sphere expansion, they can be fulfilled in terms of a late-blooming Einstein–de Sitter universe in which the deceleration parameter defined by

$$q \approx 0,\ 0 < t < t_d \text{ (radiation-dominated phase)} \tag{24}$$

and

$$q \to \frac{1}{2},\ t_d < t \to \infty \text{ (matter-dominated phase)} \tag{25}$$

causes negligible deceleration during the matter-dominated phase. It can be seen that, since the rate of change of recession velocity due to deceleration is defined by

$$\frac{dv}{dt} \approx -qHv \approx -\frac{qv}{t}, \tag{26}$$

dv/dt is only comparatively significant when t is relatively small. For $t_d \approx 10^{55}T^*$ and $T(\text{now}) \approx 10^{59}T^*$, $dv/dt \approx 0$ in comparison to what would occur in a normal Einstein–de Sitter universe when $t < t_d$. Thus, such a late-blooming Einstein–de Sitter universe can be closely approximated by a 3-sphere expanding at a constant rate.

Results given in several prior papers by the authors[19–21] indicate that the above-defined constraint set can be satisfied by expansion of a 3-sphere globally at the speed of light. The uniqueness of this condition is established by noting that free-space propagation of light along null geodesics associated with locally observed flat space-time and local adherence to the Hubble law in 3-spheres expanding at a constant rate are possible only if radial expansion (as viewed globally) occurs at the relativistic limit. Conservation of locally observed luminal flux in terms of the inverse-square law relationship found in flat space-time would also be violated if global expansion were not at the speed of light. As can be seen, this suggests then that the constraints regarding the Hubble law and conservation of luminal flux listed above are merely corollaries to the one for locally observed flat space-time.

A number of the results from the prior papers[19–21] that are relevant to the current discussion will now be cited. Given a 3-sphere expanding globally at the speed of light in 4-space, by symmetry one can define a geodesic expanding circle as a cross-section of the 3-sphere that intersects an observer and observed object pair. Let the observer be located at angular location Θ_2 on the now-surface of the expanding 3-sphere and the observed object be located at angular location Θ_1 on some past-surface of the expanding 3-sphere (at relative angle $\phi = \Theta_2 - \Theta_1$) with respect to an origin reference. Since the geodesic circle is expanding at the speed of light, the cgs space-time metric applicable to it is

$$ds^2 = c^2dt^2 - r^2d\Theta^2, \tag{27}$$

where c is the speed of light in free space and $r = ct$ is the radius of the circle. The null geodesic ($ds^2 = 0$), which intersects both observer and the observed object and along which light can propagate from object to observer, is defined by

$$rd\Theta = cdt. \tag{28}$$

Thus, the photon trajectory between object and observer lies on the shortest path between them along which light can propagate. Because $r = ct$, the trajectory intersects the expanding set of arcs of the geodesic circle and 3-sphere at a 45° angle, which implies the existence of locally observed flat space-time. The length of the photon path between object and observer in the 3-sphere is the same as what would exist in local flat space-time.[20]

The null geodesic condition given in (28) defines a photon trajectory between object and observer on the locus

$$r = c\exp(\Theta - k), t = \exp(\Theta - k), \tag{29}$$

where k is a constant of integration defining the origin reference of Θ. The expressions in (29) define a trajectory that is an outward logarithmic spiral. The path length between object and observer on this locus is defined by the metric

$$d\ell^2 = c^2[t^2 + (\exp\Theta)^2]d\Theta^2, \tag{30}$$

which yields[20]

$$\begin{aligned} \ell &= \int_{\Theta_1}^{\Theta_2} d\ell = 2^{1/2}c(\exp\Theta_2 - \exp\Theta_1) \\ &= 2^{1/2}cT_2[1 - \exp(-\phi)] \end{aligned} \tag{31}$$

as a prescription for photon path length, where T_2 is the system age of the now-surface of the 3-sphere as defined by (29). The impact of a ϕ rate of change regarding length and time-flow of the object at Θ_1 as observed at the Θ_2 locus may be defined by considering the distance to the observer relative to the object:

$$d = cT_2[1 - \exp(-\phi)] = cT_1[\exp(\phi) - 1] = r' = ct', \qquad (32)$$

where T_1 is the age of the 3-sphere at the object locus Θ_1. Taking the derivatives of r' and t' with respect to ϕ yields

$$\frac{dr'}{d\phi} = c\exp\Theta\exp\phi \qquad (33)$$

and

$$\frac{dt'}{d\phi} = \exp\Theta_1\exp\phi, \qquad (34)$$

which implies that both length and time-flow of the object as observed at Θ_2 are being enlarged by factor exp ϕ. This enlargement also causes the observed distance D of the object to be distorted as indicated by

$$D = \int_{\Theta_1}^{\Theta_2} dD = \int_{\Theta_1}^{\Theta_2} c\exp\Theta_1\exp\phi\, d\Theta = cT_2. \qquad (35)$$

This shows that the observed object is projected on the vicinity of the observer's now-surface and is moving away with recession velocity $v_r = c\phi = cz$, where $z = \phi$ radians is the observed redshift factor. Thus, it can be seen that the Hubble law is supported as required by constraint (iv), which is simply a corollary to constraints (i)–(iii). The "lens-effect" distortion implied by (30) and (31) also causes the local field of view to apparently be unbounded as $\phi \to \infty$. Thus, the locally observed scale factor corresponds to a sphere with (flat space-time) zero curvature at every stage of expansion, which is characteristic of the GRF equation solution for an Einstein–de Sitter universe. It can also be seen that as $\phi \to 0$, the actual distance $d \to cT_2\phi = D$, the observed distance. The lens-effect distortion of an observer's field of view provides an opportunity to check the validity of the hypothesis that time is strongly correlated with expansion of the 3-sphere. It can be seen that the model predicts that a 3-sphere expanding at the speed of light globally will manifest a local field of view in which the Hubble law defines the redshift of distant objects, and the brightness of distant objects at redshift z is scaled as a function of redshift by factor $\exp(-z)$. Thus, a comparison of observed redshift traces as a function of inverse-square brightness with the predictions of the topology-based model gives one indication of how well global expansion is synchronized with observed time-flow. A second basis for testing the validity of the hypothesis that time-flow and expansion are strongly correlated is the comparison of predicted age and Hubble flow of the expanding 3-sphere with what is observed. Age and Hubble flow may be predicted using the topological method under consideration by invoking some of the constraints given above.[21] Since energy is conserved globally in the expanding 3-sphere, the impact of transforming energy from radiative to matter form may be represented as a redshift factor

$$z_G + 1 = \left(\frac{\rho_m}{\rho_r}\right)^{1/3}, \rho_m \geq \rho_r, \qquad (36)$$

which is a function of the matter to radiation energy density ratio ρ_m/ρ_r. At t = now, where $\rho_m/\rho_r \approx 10^4$, $z_G \approx 20.5$ is a redshift factor that corrects globally for the portion of radiative energy converted to matter form defined by the ratio ρ_m/ρ_r.

Locally viewed, this same redshift must be defined in terms of two components, one of which is a redshift reduction in the temperature of the black-body radiator at decoupling time t_d via the Stefan–Boltzmann law to account for the directly observable impact of radiation to mass energy conversion

$$z_{\text{MBC}} + 1 = \left(\frac{\rho_m}{\rho_r}\right)^{1/4}, \rho_m \geq \rho_r. \qquad (37)$$

At t = now, where $\rho_m/\rho_r \approx 10^4$, $z_{\text{MBC}} \approx 9.0$. The other redshift component occurs as a result of photon path curvature, which is the basis for the Hubble law in the model considered here. This is defined by (29) and (35) as

$$z_{\text{PPC}} = z(T) - z(t_d) = \ln\frac{T}{t_d}. \qquad (38)$$

Thus, the sum of the two components yields an expression for the total locally observed redshift

$$z_{\text{L}} = z_{\text{MBC}} + z_{\text{PPC}} = \left(\frac{\rho_m}{\rho_r}\right)^{1/4} - 1 + \ln\frac{T}{t_d}. \qquad (39)$$

As a result of conserving luminal flux locally and nucleation at a Planck energy density level, the age of the expanding system may be determined using the inverse-square relationship

$$T = \left[\frac{\lambda_{\text{CBR}}}{(z+1)} L^* \right]^2 T^*, \qquad (40)$$

where L^* corresponds to the energy density wavelength at time T^*. The z-term in (39) is defined by the redshift correction for expressions z_G and z_L given in (36) and (39), and λ_{CBR} is the maximum energy wavelength corresponding to locally observed cosmic background radiation (CBR) temperature. Assuming a CBR temperature 2.73 K with $\lambda_{\text{CBR}} \approx 1.1 \times 10^{-1}$ cm, $L^* \approx 1.616 \times 10^{-33}$cm, $T^* \approx 5.39 \times 10^{-44}$ s, and $z_G \approx 20.5$, (37) predicts a t = now age of

$$T(\text{now}) \approx 5.40 \times 10^{17} \text{ s} \approx 17.1 \times 10^9 \text{ years}. \qquad (41)$$

It has been noted previously[21] that this corresponds to the Hubble age of an Einstein–de Sitter universe solution of the GRF equations. This is to be expected because, as shown in (35), $T = D/c\phi = D/v = H_0^{-1}$. Thus, given a strong coupling between time-flow and a speed-of-light expansion dynamic, observed system age is strictly a function of an observer's local time-flow reference H and, in general, $T = H_0^{-1}$. For the T(now) age given in (41) this implies a predicted Hubble flow value of $H_0 \approx 57.3$ km s^{-1} Mpc^{-1}.

The age solution given in (41) implies that at t = now, $z_{\text{PPC}} \approx 11.5$, the redshift locus of the CBR black-body radiator. Equation (38) indicates that this corresponds to a decoupling time locus of $t_d \approx 1.74 \times 10^5$ years. This value of t_d is required in a second expression for system age, which can be derived by generally taking $z_G = z_L$ via (36) and (39) to obtain

$$T = t_d \exp[(\rho_m / \rho_r)^{1/3} - (\rho_m / \rho_r)^{1/4}], \; \rho_m \geq \rho_r. \qquad (42)$$

Equation (42) indicates that the expanding 3-sphere defined by the constraint set manifests equal matter and radiation density levels when $T = t_d$, and that at $\rho_m/\rho_r \approx 10^4$ the predicted system age is

$$T(\text{now}) \approx 17.2 \times 10^9 \text{ years}. \qquad (43)$$

It is worth mentioning that the accuracy of the predicted values is very high in view of the fact that all the components of (40) are known with high accuracy except z. The strong agreement of (43) with (41), however, shows that both $z \approx 20.5$ and $\rho_m/\rho_r \approx 10^4$ are strongly correlated with a CBR temperature of 2.73 K. This suggests that the accuracy of the values of $z \approx 20.5$ and $\rho_m/\rho_r \approx 10^4$ is comparable to the precision used in the computation of age T(now). This is not that surprising since $\rho_m/\rho_r \approx 10^4$ is an estimate based upon knowledge of $\rho_m = \rho_c$ and the energy density implied by the current CBR temperature, which is known from COBE satellite data with an error of less than 0.4%.[27]

Agreement between the two computed values of age at T(now) in (41) and (43) also indicates the extent to which nucleation at the Planck limit is a required condition in the scenario. Such a condition allows the topological model to interface coherently with the tunneling model in the classically allowed range within the context of an inverse square based dynamic. It will be shown in a later section that this dynamic is also an important element in understanding how there can be a causal connection between time-flow and expansion. As a final note, attention is called to the fact that global expansion at a constant rate does not imply a similar locally observed dynamic. At the local level, the observed expansion rate $H_0 = T^{-1}$ appears to continually slow as time proceeds and manifests an inflationary beginning (i.e., $H_0 \to \infty$ as $T \to 0$). The spiraling photon trajectory implied by (30), which provides a basis for the Hubble law and a reduction of brightness in objects at redshift z by a factor $\exp(-z)$ can also be interpreted as implying locally observed exponential slowing in the Hubble flow rate as well as an inflationary beginning.[20] These are topological artifacts, which can be seen as a basis for a local analysis description that views the system dynamic in terms of inflationary initialization. Since the global expansion rate is constant, however, such inflation in the model used here is necessarily semi-eternal.

5. COMPARISON WITH OBSERVED DATA

As shown in the foregoing, the telltale signs to an observer at t = now in an expanding 3-sphere that indicate that time-flow is synchronized with expansion rate are

(i) the system age T(now) $\approx$ 17.1 gigayears,
(ii) the Hubble flow rate $H_0 = T^{-1} \approx 57.3$ km s^{-1} Mpc^{-1}, and
(iii) the observed brightness of distant objects at redshift z is scaled by factor $\exp(-z)$.

It is emphasized that the nature of the 3-sphere must conform to the constraint set defined at the beginning of the previous section.

The oldest known objects are the stars in globular clusters. The ages of these objects are determined by comparing stellar populations in such clusters with calibrated models. One of the best studied clusters using precise photometry methods has been M92, located near the center of the Milky Way. Analysis of the main-sequence turnoff (MSTO) attributes of the M92 population indicates that its age is 15.8 ± 2.1 gigayears.[28] If one allows an additional 1 gigayear for stellar formation, the implied age of the universe is 16.8 ± 2.1 gigayears, which compares very well with the predicted value of 17.1 gigayears. The consistency of MSTO-based age estimates has been verified by comparing such results with white dwarf age based on cooling attributes[29] and the age of open clusters in the disk of the Milky Way.[30] M92 age estimates are therefore believed to be consistent and reliable.

Measurement of the Hubble flow rate H_0 has been pursued using a variety of methods. Published results have tended to converge on two possible regions: $H_0 \approx 55 \pm 10$ and

$H_0 \approx 80 \pm 10$. The actual range of H_0 in particular reports is frequently smaller than ± 10, but the size of the gap between the two regions of convergence suggests the presence of unexplained impacts and the need for further investigation. Some of the methods used to obtain results that support the lower region concern evaluations of

(i) the distance to galaxy IC4182 using SN1937C as a standard candle, where $H_0 \approx 54 \pm 8$ (via the Hubble telescope),[31,32]
(ii) the brightness of SN1937C using the standard model for type Ia supernovae with $H_0 \approx 60 \pm 10$[33] and $H_0 \approx 50 \pm 9$[34], and
(iii) the Sunyaev–Zel'dovich distortion effect on CBR with $H_0 \approx 52 \pm 22$[35] and $H_0 \approx 55 \pm 17$.[36]

Methods that support the upper region involve evaluations of

(i) the velocity gradient along a luminous arc in cluster A2390 using the Tully–Fisher method ($q_0 = 1/2$) with $H_0 \approx 75 \pm 20$,[37]
(ii) Tully–Fisher (TF) distance ladders based on galaxy brightness as a function of rotation speed,[38]
(iii) surface brightness fluctuation (SBF) in galaxies as a function of statistical brightness distributions,[39] and
(iv) luminosity cutoff attributes of planetary nebulae (PNLF).[40]

These methods depend upon an absolute calibration that makes use of Cepheid variable star distances, supernovae type Ia standard candles, or data fit to an expanding photosphere model.[33] Recent recession velocity data for Cepheids in the Coma and Virgo clusters via the Hubble telescope give a value of $H_0 \approx 80 \pm 17$.[41]

Since these measurements avoid atmospheric interference, they are reviewed as more reliable than prior Earth-based Cepheid measurements, which were used to calibrate TF, SBF, and PNLF estimates of H_0. It can be seen that the predicted value of H_0 for expansion-synchronized time-flow corresponds strongly to the lower region of convergence mentioned above. Thus, if the lower range is correct, this constitutes strong evidence in favor of expansion-synchronized time-flow. Validity of the upper range, however, would testify to the contrary.

Several recent galaxy-based redshift-magnitude surveys measure galaxy brightness as a function of redshift to test the linearity of the Hubble law.[42] One of these is a survey of field galaxies, which deals with luminosity scatter statistically and corrects for changing luminosity as a function of evolution.[43] An evaluation of the measured Hubble law curve for this survey shows that at average redshift $z = 1$, brightness reduction is approximately 2.7 and at average $z = 0.45$, brightness is reduced by approximately 1.7. Thus, the measured redshift-brightness curve strongly supports a prediction that the observed brightness of objects at redshift z will be scaled by a factor $\exp(-z)$. A second survey regards radio galaxies with high intrinsic luminosity in the K-band, which does not correct for evolving changes in galaxy luminosity.[44]

The results of this survey show that brightness at redshift z is diminished approximately by a factor $[\exp(-z)]/2$. It is generally believed, however, that such radio galaxies were more luminous at the earlier stages of their existence than at low redshift.[41] Thus, because a standard luminosity baseline is assumed, the survey results for Hubble law deviation are partially masked by evolutionary changes in radio galaxy luminosity. The second survey also suffers from the use of a relatively small sample size in comparison to the first and makes no use of statistical averaging.

6. EXPANSION-BASED MASTER ARROW OF TIME ARGUMENT

A second question, which follows naturally from an evaluation of how well system expansion and time-flow are synchronized, regards the existence of a causal connection between the two. The argument offered here is focused on the way in which the inverse-square distance dynamic of phenomena seen locally in the flat space-time of an expanding 3-sphere can be mechanistically dependent on the global volumetric expansion dynamic of the system. Special relativity is invoked as a basis for establishing the link to a causal relationship. A description of a static form of the 3-sphere geometry under consideration here may be found in the recently updated text on gravitation by Ohanian and Ruffini.[45] The point under consideration is CBR energy propagation, and the question to be answered is what does it imply about Maxwell's equations in the local framework? It can be seen that the expression for system age in (37) depends upon the conservation of luminous flux. This is based upon the inverse-square formula for plane-wave spreading in Lorentz-invariant flat space-time:

$$\frac{\lambda_{\text{CBR}}}{r_{\text{init}}^2} = (z+1)\frac{L^*}{r^2} = \frac{z+1}{L^*}, \tag{44}$$

where r is the distance from an observer to the location of the CBR source. The form of (33), however, indicates that conserved energy is spreading volumetrically at the global level. As defined in Section 4, this volumetric spreading is due to the speed-of-light expansion of the 3-sphere and implies that the global energy density wavelength λ_G has a form

$$\lambda_G = k_1(z+1)r^3, \tag{45}$$

where k_1 is a constant. Due to the Lorentz-invariance of the observer's local framework and a speed-of-light expansion dynamic globally, however, the form of wavelength λ_G is altered when observed locally. The length in the direction of the observer's line of sight contracts to zero and the locally observed form of (45) is transformed to one like that given in (44). A set of observers on the locus of a sphere at distance r will view the global wavelength λ_G as having local wavelength

$$\lambda_L = k^2(z+1)r^2. \tag{46}$$

As $r \to L^*$, $\lambda_L \to (z+1)L^*$. Thus, $k_2 = L^{*^{-1}}$ and $\lambda_L = \lambda_{\mathrm{CBR}}$. This illustrates, then, how locally observed CBR plane-wave spreading is causally dependent on volumetric spreading at the global level. Indeed, both the inverse-square form and the time-flow dynamic of the expression in (44) are a consequential reflection of the expansion dynamic and geometry at the global level. Moreover, if the volumetric spreading were to halt, the observed form of plane-wave spreading would not only lose the basis of its dynamic but also undergo an alteration of its inverse-square law form to one that reflects the static condition of the global state. Such a static state therefore also implies a necessary violation of the luminous flux conservation constraint defined in Section 4. Thus, a static global state for the 3-sphere considered here is not compatible with the existence of Maxwell's equations in the local framework of such a system.

The foregoing argument demonstrates that conservation of luminous flux is a consequence of a Lorentz contraction in the observer's local flat space-time framework of a 3-sphere that is expanding at light speed and conserving energy. Thus, constraint number (v) (in Section 4) is a corollary to constraints (i)–(iii) with the additional caveat that the 3-sphere be expanding at light speed. Such an expansion dynamic is a necessary attribute to support CBR wave propagation in the local framework per Maxwell's equations. The need for some form of continuing semi-eternal inflation, which cancels the effects of locally perceived gravitational drag on expansion, is therefore also implied. It is also possible to identify some attributes of alternative dynamics to the light-speed expansion form considered here. If the global expansion dynamic were slower than light speed, there would be no locally perceived conservation of CBR flux. Lack of light-speed motion along the line of sight would not support the null-level contraction cited above. On the other hand, if the 3-sphere were contracting at light speed, a local observer would perceive the CBR source seen in the case of expansion instead as a sink. This implies that the time dynamic is reversed in a spatially contracting environment. Conservation of luminous flux per Maxwell's equations would remain intact, but radiation propagation would be perceived in terms of an inward wave dynamic rather than the outward one seen in an expanding system.

7. CONCLUSION

The foregoing illustrates how the quantum cosmology tunneling model can be adapted to accommodate a scenario that features nucleation at the Planck limit and a late-blooming dual-phase form of an Einstein–de Sitter universe that is expanding globally at a constant rate. Such a model supports application of the WKB semiclassical approximation method and leads to wave-function solution forms that are coherent with prior tunneling results requiring nucleation at maximum energy density. The resulting wave-function forms are also coherent with what one would expect classically for such a scenario. The topology of a Friedmann–Lemaître based 3-sphere model is exploited analytically to obtain predicted values in the classically allowed range for system age, Hubble flow rate, and Hubble law brightness deviation. These predicted results are shown to be in good agreement with actual observations.

The rather strong agreement between predicted values and observed data testifies favorably in support of the notion that observed time-flow is expansion synchronized. This result can be interpreted in terms of a late-blooming Einstein–de Sitter universe that expands with negligible gravitational drag. Such a view implies that matter has not formed quickly enough to have a significant impact on the expansion dynamic. Alternatively, one could also attribute the outcome to the existence of some form of semi-eternal inflation, which acts to nullify the impact of gravitational drag. Observations that indicate a total absence of such drag would tend to support this viewpoint. The current lack of a well-focused set of measurements on the Hubble flow rate, however, prevents one from reaching an immediate conclusion in the matter.

The argument regarding a causal link between 3-sphere expansion and the observed time-flow dynamic and geometric form of the CBR wave propagation is compelling, but not sufficiently complete to be conclusive. If the expansion does not occur at light speed as suggested in the argument, the alternative appears to be a setting in which Maxwell's equations are reduced to a crude approximation. Thus, the weight of the argument appears to lend support to a zero gravitational drag semi-eternal inflation viewpoint. It also motivates one to search for further evidence that will strengthen (or weaken) the case for a causal link. Clearly, it would be useful to have additional arguments relating to quantum-state and relativity-based impacts that are attributable to the 3-sphere expansion dynamic.

If a causal link exists and the expansion dynamic were not restricted to a constant rate, it would have serious implications regarding observability. In general, local observations force one to view the time-flow attributes of a distant object in terms of time-flow in the local framework. Thus, if a causal link between time-flow and expansion were not bound to a fixed expansion rate, a real Einstein–de Sitter universe could appear instead to have a late-blooming form like that discussed in this paper. The actual revolutionary dynamic would be unobservable. This possibility is avoided by imposing a constraint that demands that 3-sphere expansion be at light speed. The speed of light is thereby forced to be synonymous with the speed of time.

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Résumé

Une version modifiée du modèle semi-classique du modèle d'effet tunnel tridimensionnel est appliquée à un scénario de cosmologie quantique caractérisé par une nucléation à la limite de densité d'énergie de Planck, et par une phase dominée par la radiation se transformant lentement en une phase dominée par la matière. Dans les limites classiques permises, la géométrie sphérique fermée du modèle, est contraint topologiquement pour définir un univers Einstein-de-Sitter "tardif" avec une perception locale d'un espace-temps plat et une expansion synchronisée du temps. Les attributs de l'écoulement du temps, prédits par ce modèle sont alors utilisés comme base, pour évaluer comment les âges, l'écoulement Hubble et la déviation Hubble sont bien observés en fonction de la luminosité, par rapport au modèle. Il est démontré que malgré la concordance importante entre les données observées et les prédictions du modèle soutenant le point de vue que l'écoulement du temps est synchronisé en l'expansion, une amélioration de la résolution de la mesure de la vitesse d'écoulement Hubble observée est nécessaire pour arriver à une conclusion définitive. Le niveau du procédé du lien entre l'expansion tridimensionnelle et le déploiement de l'onde plane CBR observée localement, est établi en fonction d'une transformation causée par l'expansion de l'espace temps. Ceci démontre comment l'expansion tridimensionnelle peut servir de base causale pour la dynamique de la "flèche du temps", vue localement dans la propagation des champs électromagnétiques.

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